Self-similarity and the pair velocity dispersion.

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ABSTRACT

We have considered linear two point correlations of the form $\frac{1}{x^{\gamma}}$ which are known to have a self-similar behaviour in a $\Omega = 1$ universe. We investigate under what conditions the non-linear corrections, calculated using the Zel'dovich approximation, have the same self-similar behaviour. We find that the scaling properties of the non-linear corrections are decided by the spatial behaviour of the linear pair velocity dispersion and it is only for the cases where this quantity keeps on increasing as a power law (i.e. for $\gamma < 2$) do the non-linear corrections have the same self-similar behaviour as the linear correlations. For ($\gamma > 2$) we find that the pair velocity dispersion reaches a constant value and the self-similarity is broken by the non-linear corrections. We find that the scaling properties calculated using the Zel'dovich approximation are very similar to those obtained at the lowest order of non-linearity in gravitational dynamics and we propose that the scaling properties of the non-linear corrections in perturbative gravitational dynamics also are decided by the spatial behaviour of the linear pair velocity dispersion.

Subject headings: Galaxies: Clustering - Large Scale Structure of the Universe methods: analytical

1. Introduction.

The equations governing the evolution of the statistical properties of disturbances in a critical ($\Omega = 1$), matter dominated universe are known to admit self-similar solutions (Peebles 1980). This is because the universe expands as a power law of time i.e. $a(t) \propto t^{\frac{2}{3}}$ and gravity itself does not introduce any preferred scale. As a consequence of this it is possible, using the linear theory of density perturbations, to construct correlation functions which have a self-similar behaviour over a range of scales. This method is valid only as long as the correlations on these scales are extremely small. On the other hand it is possible to use the stable clustering assumption to construct correlation functions that have a self-similar behaviour over a range of scales. The

latter assumption is valid only on small scales where virialized objects have already formed and clustering in real space has ceased to increase. On scales where neither of these assumptions can be used very little is known.

Hamilton et. al. (1991) have suggested a universal scaling relation for the two point correlation function based on numerical evidence from N-body simulations. In more recent papers Nityananda & Padmanabhan (1994), Peacock & Dodds (1994), Jain et. al. (1995), Lokas et.al. (1995) and Padmanabhan (1996) have investigated the proposed universal scaling relations. In an earlier paper (Bharadwaj 1996) we have studied the lowest order non-linear corrections to the two point correlation function for cases where the linear correlation function has a self-similar behaviour over a range of scales. We restricted ourselves to cases where the linear power spectrum has the form $P(k) \propto k^n$ with $n \geq 0$ at small k and we found that the non-linear corrections do not have the same self-similar behaviour as the linear two point correlation function. In a recent paper Scoccimarro and Frieman (1996) have addressed the same question for initial conditions which include $n < 0$. For $n \geq 0$ their conclusions are similar to ours, but they find that for $n < -1$ the lowest order non-linear correction has the same self-similar behaviour as the linear power spectrum.

In this paper we consider initial perturbations which are a random Gaussian field and have a two point correlation function which is self-similar over a range of scales. We study the evolution of the correlation function analytically using the Zel'dovich approximation (ZA) (Zel'dovich 1970) and we investigate under what general conditions the non-linear corrections to the two point correlation function exhibit the same self-similar behaviour as the linear two point correlation function. ZA is known to be a very good approximation to the full gravitational dynamics in the weakly non-linear regime before the effects of multi-streaming become important. It is thus expected that the results of this investigation should also hold for gravitational dynamics and we have compared our results with the results available from perturbative non-linear gravitational dynamics (GD). The use of ZA instead of GD makes the algebra much more tractable and it is much simpler to interpret the results. It is also hoped that the results of this investigation may help in building up correlation functions that are self-similar in the non-linear epoch starting from correlations that are self-similar in the linear regime.

The evolution of the various correlation functions in ZA has been studied by earlier authors including Bond & Couchman (1988), Grinstein & Wise (1987), and Schneider & Bartlemann(1995). In this paper we use the notation and some of the results of our earlier paper (Bharadwaj 1995) where we also presented some of our preliminary results regarding the non-linear corrections to the two point correlation function in ZA and a comparison with the results from GD.

2. The two point correlation in ZA.

For a $\Omega = 1$ universe ZA defines a map

$$x_\mu(t) = x_\mu(t_0) + a(t)u_\mu \tag{1}$$

from the initial position $x_\mu(t_0)$ of a particle to its position $x_\mu(t)$ at some later time t in a comoving coordinate system. Here the subscript μ takes values $1, 2$ and 3 corresponding to the three Cartesian components and the Einstein summation convention holds for it. The quantity u_μ is related to the peculiar velocity of the particle and we shall refer to it as the velocity.

We are interested in the evolution of the statistical properties of an ensemble of systems whose evolution is governed by ZA. In all the members of the ensemble the particles are all initially uniformly distributed and the initial velocity field is assumed to be irrotational. It is also assumed that the velocity field in any member of the ensemble is a particular realization of a Gaussian random field. These initial conditions can be fully specified by the velocity-velocity correlation which can be written in terms of a potential $\phi(x)$ as

$$< u_\mu(x^1)u_\nu(x^2) >= -\partial_\mu\partial_\nu\phi(x) \tag{2}$$

where $\partial_\mu = \frac{\partial}{\partial x_\mu}$ and

$$x =\mid \vec{x}^1 - \vec{x}^2 \mid \ . \tag{3}$$

For such an ensemble the two point correlation function can be written in a perturbative expansion as (Bharadwaj 1995)

$$\begin{aligned} \xi(x,t) \ &= \ \Sigma_{n=1}^{\infty} a^{2n} \frac{1}{n!} \partial_{\mu_1}\partial_{\nu_1}\partial_{\mu_2}\partial_{\nu_2}...\partial_{\mu_n}\partial_{\nu_n} \left[\left(\partial_{\mu_1}\partial_{\nu_1}\phi(x) - \frac{1}{3}\delta_{\mu_1\nu_1}\nabla^2\phi(0) \right) \right. \\ & \left. \left(\partial_{\mu_2}\partial_{\nu_2}\phi(x) - \frac{1}{3}\delta_{\mu_2\nu_2}\nabla^2\phi(0) \right) ... \left(\partial_{\mu_n}\partial_{\nu_n}\phi(x) - \frac{1}{3}\delta_{\mu_n\nu_n}\nabla^2\phi(0) \right) \right] \ . \end{aligned} \tag{4}$$

The quantity that appears in the right hand side of this equation is the dispersion of the pair velocity (i.e. the difference between the velocity at the point x^2 and the point x^1) which is defined as

$$\begin{aligned} < v_\mu v_\nu > (x) \ &= \ < (u_\mu(x^2) - u_\mu(x^1))(u_\mu(x^2) - u_\mu(x^1)) > \\ &= \ 2\left(\partial_{\mu_2}\partial_{\nu_2}\phi(x) - \frac{1}{3}\delta_{\mu_2\nu_2}\nabla^2\phi(0) \right) \ . \end{aligned} \tag{5}$$

This quantity which we shall refer to the as the pair velocity dispersion is related to the dispersion of the relative peculiar velocities $\sigma_{\mu\nu}(x)$ in the linear epoch, and we have

$$\sigma_{\mu\nu}(x,t) = \left(a(t)\frac{da(t)}{dt} \right)^2 < v_\mu v_\nu > (x) \tag{6}$$

This dispersion arises due to the spread in the relative velocities across the various realizations in the ensemble. The pair velocity dispersion is a symmetric tensor and because the initial conditions are statistically homogeneous and isotropic, it can in general be written as

$$< v_\mu v_\nu > (x) >= \delta_{\mu\nu}P(x) + \frac{x_\mu x_\nu}{x^2}Q(x) \tag{7}$$

where $P(x)$ is the dispersion of the relative velocity component perpendicular to the separation $\vec{x}(=\vec{x}^2-\vec{x}^1)$ and $P(x)+Q(x)$ is the dispersion in the velocity component parallel to $\vec{x}$.

The two point correlation function can be written in terms of the pair velocity dispersion as

$$\xi(x,t)=\Sigma_{n=1}^{\infty}\frac{1}{n!}\left(\frac{a^2}{2}\right)^n \qquad \partial_{\mu_1}\partial_{\nu_1}\partial_{\mu_2}\partial_{\nu_2}...\partial_{\mu_n}\partial_{\nu_n}\left[<v_{\mu_1}v_{\nu_1}>(x)<v_{\mu_2}v_{\nu_2}>(x)\right.$$
$$\left.... <v_{\mu_n}v_{\nu_n}>(x)\right]\,. \tag{8}$$

From this we obtain the linear two point correlation function as

$$\xi^{(1)}(x,t)=a^2\xi^{(1)}(x)=\frac{a^2}{2}\partial_{\mu_1}\partial_{\nu_1}<v_{\mu_1}v_{\nu_1}>(x)=a^2\nabla^4\phi(x)\,. \tag{9}$$

Given the linear two point correlation function we can invert equation (9) and express the initial pair velocity dispersion in terms of the linear two point correlation function as

$$\begin{aligned}<v_\mu v_\nu>(x)=&\frac{2}{3}\delta_{\mu\nu}\int_0^x\xi^{(1)}(y)ydy\\ &-\partial_\mu\partial_\nu(x)\int_0^x\xi^{(1)}(y)y^2dy\\ &-\frac{1}{3}\partial_\mu\partial_\nu(1/x)\int_0^x\xi^{(1)}(y)y^4dy\end{aligned} \tag{10}$$

and finally we are in a position to start investigating the nature of the non-linear corrections to the two point correlation function, given the linear two point correlation function.

We consider situations where the linear two point correlation function at some large scales i.e. for $x>x_1$ has the form

$$\xi^{(1)}(x,t)=a^2(t)\left(\frac{x_0}{x}\right)^\gamma=\left(\frac{x_0(t)}{x}\right)^\gamma \tag{11}$$

where

$$x_0(t)=x_0a(t)^{\frac{2}{\gamma}}=x_0\left(\frac{t}{t_0}\right)^{\frac{4}{3\gamma}}\,. \tag{12}$$

We see that the two point correlation function has a self-similar behaviour over a range of scales in the linear epoch and the effect of temporal evolution is to just scale the length scale $x_0(t)$ that appears in the two point correlation function. We want to investigate under what conditions the non-linear corrections to the two point correlation function has the same self-similar behaviour. Since the non-linear corrections are determined by the pair velocity dispersion, we proceed by first investigating the behaviour of the pair velocity dispersion.

3. The pair velocity dispersion.

In this section we investigate the spatial behaviour of the pair velocity dispersion for different values of the index γ. We separately consider the three different moments of the linear two point correlation function that appear in equation (10) for the dispersion of the pair velocity

$$A(x) = \int_0^x \xi^{(1)}(y) y dy \tag{13}$$

$$B(x) = \frac{1}{x} \int_0^x \xi^{(1)}(y) y^2 dy \tag{14}$$

$$C(x) = \frac{1}{x^3} \int_0^x \xi^{(1)}(y) y^4 dy \tag{15}$$

It should be noted that all three of the functions defined above have dimension L^2 and it is possible that any of them may introduce a new length scale in the evolution.

3.1. The behaviour of A.

For $\gamma > 2$ the integral in A converges in the limit $x \to \infty$ and at large x we have

$$A(x) = \int_0^\infty \xi^{(1)}(y) y dy + \left(\frac{x_o^\gamma}{2-\gamma} \right) \frac{1}{x^{\gamma-2}} \,. \tag{16}$$

Here the first term is a constant and the second term decays as x increases, and at large x $A(x)$ tends to a constant value..

At the value $\gamma = 2$ we have

$$A(x) = \int_0^{x_1} \xi^{(1)}(y) y dy + x_0^2 \ln \left(\frac{x}{x_1} \right) \tag{17}$$

For $\gamma < 2$ we have

$$A(x) = \int_0^{x_1} \xi^{(1)}(y) y dy - \left(\frac{x_o^\gamma}{2-\gamma} \right) x_1^{2-\gamma} + \left(\frac{x_o^\gamma}{2-\gamma} \right) x^{2-\gamma} \tag{18}$$

where the first two terms are constants and the last term increases monotonically with x and for large x we have

$$A(x) = \left(\frac{x_o^\gamma}{2-\gamma} \right) x^{2-\gamma} \tag{19}$$

3.2. The behaviour of B

For $\gamma > 3$ the integral in B converges as $x \to \infty$ and at large x we have

$$B(x) = \frac{1}{x}\int_0^{\infty} \xi^{(1)}(y) y^2 dy + \left(\frac{x_o^{\gamma}}{3-\gamma}\right)\frac{1}{x^{\gamma-2}} \,. \tag{20}$$

Here the first term falls as $\frac{1}{x}$ and the second term falls off faster than this, and hence at large x the first term dominates.

At the value $\gamma = 3$ we have

$$B(x) = \frac{1}{x}\left[\int_0^{x_1} \xi^{(1)}(y) y dy + x_0^3 \ln\left(\frac{x}{x_1}\right)\right] \tag{21}$$

and for large x we have

$$B(x) = \frac{x_0^3}{x}\log\left(\frac{x}{x_1}\right) \tag{22}$$

and for $\gamma < 3$ we have

$$B(x) = \frac{1}{x}\left(\int_0^{x_1} \xi^{(1)}(y) y^2 dy - \left(\frac{x_o^{\gamma}}{3-\gamma}\right) x_1^{3-\gamma}\right) + \left(\frac{x_o^{\gamma}}{3-\gamma}\right) x^{2-\gamma} \tag{23}$$

where for large x the first terms falls off as $\frac{1}{x}$ and the second term falls off slower. Thus at large x we have

$$B(x) = \left(\frac{x_o^{\gamma}}{3-\gamma}\right) x^{2-\gamma} \,. \tag{24}$$

where this is a decaying function for $2 < \gamma < 3$, it is a constant for $\gamma = 2$ and it is a monotonically increasing function for $\gamma < 2$.

3.3. The behaviour of C

For $\gamma > 5$ the integral in C converges as $x \to \infty$ and at large x we have

$$C(x) = \frac{1}{x^3}\int_0^{\infty} \xi^{(1)}(y) y^4 dy + \left(\frac{x_o^{\gamma}}{5-\gamma}\right)\frac{1}{x^{\gamma-2}} \,. \tag{25}$$

here the first term falls as $\frac{1}{x^3}$ and the second term falls off faster than this, and hence at large x the first term dominates.

At the value $\gamma = 5$ we have

$$C(x) = \frac{1}{x^3}\left[\int_0^{x_1} \xi^{(1)}(y) y^4 dy + x_0^5 \ln\left(\frac{x}{x_1}\right)\right] \tag{26}$$

and for large x we have

$$C(x) = \frac{x_0^5}{x^3}\ln\left(\frac{x}{x_1}\right) \tag{27}$$

For $\gamma < 5$ we have

$$C(x) = \frac{1}{x^3}\left(\int_0^{x_1} \xi^{(1)}(y)y^4 dy - \left(\frac{x_o^\gamma}{5-\gamma}\right) x_1^{5-\gamma}\right) + \left(\frac{x_o^\gamma}{5-\gamma}\right) x^{2-\gamma} \tag{28}$$

where at large x the first terms falls off as $\frac{1}{x^3}$ and the second term dominates. Thus at large x we have

$$C(x) = \left(\frac{x_o^\gamma}{5-\gamma}\right) x^{2-\gamma} . \tag{29}$$

where this is a decaying function for $2 < \gamma < 3$, it is a constant for $\gamma = 2$ and it is a monotonically increasing function for $\gamma < 2$.

3.4. The spatial behaviour of the dispersion of pair velocities.

Putting together the various components calculated earlier we can now write expressions for the dispersion of the pair velocity for different values of the index γ.

For the cases where $\gamma < 2$ the behaviour is very simple and we have

$$\begin{aligned} < v_\mu v_\nu > (x) = \quad & \delta_{\mu\nu}\left(\frac{2}{3(2-\gamma)} - \frac{1}{(3-\gamma)} + \frac{1}{3(5-\gamma)}\right) x_o^\gamma x^{2-\gamma} \\ + \quad & \frac{x_\mu x_\nu}{x^2}\left(\frac{1}{3-\gamma} - \frac{1}{5-\gamma}\right) x_o^\gamma x^{2-\gamma} \end{aligned} \tag{30}$$

For this case we see that both the components of the pair velocity dispersion increase as $\propto x^{2-\gamma}$.

Next, for $\gamma = 2$ we get

$$< v_\mu v_\nu > (x) = \frac{2}{3} x_0^2 \left[\delta_{\mu\nu} \ln\left(\frac{x}{x_1}\right) + \frac{x_\mu x_\nu}{x^2}\right] . \tag{31}$$

Here both the components of the pair velocity dispersion increase as $\ln(x)$ and are nearly equal as we go to large separations

For $2 < \gamma < 3$ we have to add an extra term

$$\delta_{\mu\nu} l^2 = \frac{2}{3}\delta_{\mu\nu}\int_0^\infty \xi^{(1)}(y) y dy \tag{32}$$

to equation (30). This term dominates the behaviour of the pair velocity dispersion at large x as the contribution from the terms in equation (30) get smaller as x increases and both the components of the pair velocity dispersion tend to a constant value l^2. Thus we see that we have one new length scale now i.e. l and this plays a very crucial in the later discussion.

For $\gamma = 3$ we have

$$< v_\mu v_\nu > (x) = \frac{2}{3}\delta_{\mu\nu}\int_0^\infty \xi^{(1)}(y) y dy - \delta_{\mu\nu}\frac{x_0^3}{x}\left[\frac{1}{2} + \ln\left(\frac{x}{x_1}\right)\right] \\ -\frac{x_\mu x_\nu}{x^2}\frac{x_0^3}{x}\left[\frac{1}{2} - \ln\left(\frac{x}{x_1}\right)\right] . \tag{33}$$

For $3 < \gamma < 5$, in addition to the expression (32), we have to also add

$$\left[\frac{x_\mu x_\nu}{x^2} - \delta_{\mu\nu}\right]\frac{1}{x}\int_0^\infty \xi^{(1)}(y) y^2 dy \tag{34}$$

to equation (30).

For $\gamma = 5$ we have

$$< v_\mu v_\nu > (x) = \frac{2}{3}\delta_{\mu\nu}\int_0^\infty \xi^{(1)}(y) y dy + \left[\frac{x_\mu x_\nu}{x^2} - \delta_{\mu\nu}\right]\frac{1}{x}\int_0^\infty \xi^{(1)}(y) y^2 dy \\ +\delta_{\mu\nu}\frac{x_0^5}{3x^3}\left[\frac{5}{6} + \ln\left(\frac{x}{x_1}\right)\right] - \frac{x_\mu x_\nu}{x^2}\frac{x_0^5}{x^3}\left[\frac{1}{2} + \ln\left(\frac{x}{x_1}\right)\right] \tag{35}$$

For $\gamma > 5$, in addition to the two terms in expressions (32) and (34) we have to also add

$$\left[\frac{1}{3}\delta_{\mu\nu} - \frac{x_\mu x_\nu}{x^2}\right]\frac{1}{x^3}\int_0^\infty \xi^{(1)}(y) y^4 dy \tag{36}$$

to equation (30).

Finally, we see that for $\gamma < 2$ the dispersion in the pair velocity keeps on increasing as a power law. For $\gamma = 2$ it increases logarithmically with the separation and for all $\gamma \leq 2$ pair velocity dispersion tends to infinity as the separation x tends to infinity. For $\gamma > 2$ we find that the pair velocity dispersion reaches the constant value l^2 which is determined by the two point correlation at small separations.

We next consider some specific cases where the linear power spectrum is of the form $P(k) = Ae^{-k}k^n$ with $n = (a) - 2, (b) - 1, (c)\,0$ and $(d)\,1$. The corresponding values of γ are $(a)\,1, (b)\,2, (c)\,4$ and $(d)\,4$. For the case with $n = 0$ the large x behaviour is decided by the exponential cutoff in $P(k)$ and because of this it has the same power law index ($\gamma = 4$) as the $n = 1$ case. Figure (1) shows the tangential component of the pair velocity dispersion for these four cases and it illustrates the main point of this section. The behaviour of the radial component of the pair velocity dispersion is very similar.

4. The non-linear corrections to ξ.

Here we use the pair velocity dispersion calculated in the previous section to investigate the nature of the non-linear corrections to the two point correlation function. From equation (10) we obtain the lowest order non-linear correction to the two point correlation function as

$$\xi^{(2)}(x,t) = a^4 \xi^{(2)}(x) = \frac{a^4}{8} \partial_{\mu_1} \partial_{\nu_1} \partial_{\mu_2} \partial_{\nu_2} \left[< v_{\mu_1} v_{\nu_1} > (x) < v_{\mu_2} v_{\nu_2} > (x) \right] . \tag{37}$$

For $\gamma < 2$ the lowest order non-linear correction to the two point correlation function is

$$\xi^{(2)}(x) = \frac{2(285 - 679\gamma + 611\gamma^2 - 259\gamma^3 + 52\gamma^4 - 4\gamma^5)}{(2-\gamma)(3-\gamma)^2(5-\gamma)^2} \left(\frac{x_0}{x} \right)^{2\gamma} . \tag{38}$$

and for this range of γ we find that $\xi^{(2)}(x,t) \propto \left[\xi^{(1)}(x,t) \right]^2$. For $\gamma < 2$ the pair velocity dispersion is a power law in x and it is $\sim x^{2-\gamma}$. To calculate the n^{th} order non-linear corrections to the two point correlation function we take n of these functions i.e. an expression $\propto (x^{2-\gamma})^n$ and we act on this with $2n$ spatial derivatives and the result is $x^{-n\gamma} \propto \left[\xi^{(1)}(x) \right]^n$. Thus we see that nth order non-linear correction is of the form $\xi^{(n)}(x) \propto [\xi^{(1)}(x)]^n$ and we can write the two point correlation function in a series of the form

$$\xi(x,t) = \sum_1^{\infty} c_n(\gamma) \left[\xi^{(1)}(x,t) \right]^n \tag{39}$$

where the $c_n(\gamma)$ are coefficients which depend only on γ. We see that for $\gamma < 2$ the non-linear corrections to the two point correlation function have the same self-similar behaviour as the linear two point correlation function.

For $\gamma = 2$ we have

$$\xi^{(2)}(x) = \frac{2}{3} \left(\frac{x_o}{x} \right)^4 \ln \left(\frac{x}{x_1} \right) . \tag{40}$$

For this case we find that the non-linear correction cannot be written in terms of the linear two point correlation alone and it does not have a self-similar behaviour.

These are the only cases where the pair velocity dispersion keeps on increasing at large x. For larger values of γ the pair velocity dispersion reaches a constant value and as consequence the behaviour of the two point correlation too is different. For all higher values of γ, in addition to the contribution to $\xi^{(2)}(x)$ given by equation (38) which behaves as $x^{(4-2\gamma)}$, there are other terms which fall of slower and dominate the behaviour at large x. For $(2 < \gamma < 3)$ we have

$$\xi^{(2)}(x) = \left[\frac{1}{3} \int_0^{\infty} \xi^{(1)}(y) y dy \right] \gamma(\gamma - 1) \frac{x_0^{\gamma}}{x^{2+\gamma}} . \tag{41}$$

For these cases we see that the self-similar behaviour is broken. This is because the pair velocity dispersion reaches a constant value and this introduces a new length scale l whose square appears in the square brackets in equation (41). This length scale l changes the scaling property of the two point correlation function.

For $\gamma > 3$ we have to take into account one more term and we have

$$\xi^{(2)}(x,t) = \left[\frac{1}{3}\int_0^\infty \xi^{(1)}(y) y dy\right] \gamma(\gamma-1)\frac{x_0^\gamma}{x^{2+\gamma}} + \frac{3}{x^6}\left[\int_0^\infty \xi^{(1)}(y) y^2 dy\right]^2 . \tag{42}$$

For $\gamma < 4$ it si obvious that the first term falls off slower than x^{-6} and it dominates the large x behaviour. For $\gamma = 4$ both the terms have a x^{-6} behaviour and we cannot drop any one of them. For $\gamma > 4$ the first term falls off faster than x^{-6} and we would expect the second term to dominate, but for many of the cases of interest the integral $\int_0^\infty \xi^{(1)}(y)y^2dy$ is zero and then it is only the first term that contributes. This is illustrated by the two cases (c) and (d) that we have considered earlier. The moments of the two point correlation function can be written in terms of the power spectrum as

$$\int_0^\infty \xi^{(1)}(y) y dy = \frac{1}{2\pi^2}\int_0^\infty P(k) dk \tag{43}$$

and

$$\int_0^\infty \xi^{(1)}(y) y^2 dy = \frac{P(0)}{4\pi} . \tag{44}$$

For both (c) and (d) we have γ=4, but using the above equations we find that for (c) $(i.e.n = 0)$ we obtain $\int_0^\infty \xi^{(1)}(y)y^2dy = \frac{A}{4\pi}$ and for (d) $(i.e.n = 1)$ we have $\int_0^\infty \xi^{(1)}(y)y^2dy = 0$, and for both of them we have $\int_0^\infty \xi^{(1)}(y)ydy = \frac{A}{2\pi^2}$. Thus for (c) the second term in equation (42) contributes and for (d) it is zero and for these two cases we obtain

$$\text{for (a)} \qquad \xi^{(2)}(\mathrm{x}) = \frac{\mathrm{A}}{\pi^2}\left(2\mathrm{x}_0^4 + \frac{3\mathrm{A}}{16}\right)\frac{1}{\mathrm{x}^6} \tag{45}$$

and

$$\text{for (b)} \qquad \xi^{(2)}(\mathrm{x}) = \frac{2\mathrm{A}\mathrm{x}_0^4}{\pi^2}\frac{1}{\mathrm{x}^6} . \tag{46}$$

Finally we can generalize this to say that for all cases where linear power spectrum is of the form $P(k) \propto k^n$ with $n > -1$ for small k, the lowest order non-linear correction to the two point correlation function is given by equation (41), except for the case when $n = 0$. For $n = 0$ we have to take into account the extra term in equation (42).

In a recent paper Taylor and Hamilton (1996) have considered the evolution of the non-linear power spectrum in the Zel'dovich approximation. They present exact analytic results for cases where the linear power spectrum is a power law with the power law index $n = -2, -1$ i.e. $(\gamma = 1, 2)$. We have compared our result at the lowest order of non-linearity with the corresponding result implied by the exact expression calculated by Taylor and Hamilton (1996). We find that while the two match for $n = -2$, there is a disagreement for $n = -1$. This is because for $n = -1$ $(\gamma = 2)$ they have considered the pair velocity dispersion to bee a constant value whereas we find that it has a logarithmic dependence on the separation. In addition to this, they have considered $n = -1$ as a limiting case of a situation where the power law index is of the form $n = -1 - \epsilon$. In our study here we find that the behaviour of the case with $\gamma = 2$ i.e. $n = -1$ is quite different from the results for $\gamma < 2$ and we do not expect that the limit taken by Taylor and Hamilton (1996) will give the correct result for $n = -1$.

5. Discussion and Conclusion.

We have considered linear two point correlation functions that have the form $\frac{1}{x^\gamma}$ at large separations. For $\Omega = 1$ they have a self-similar behaviour in the linear epoch. We have investigated under what conditions the non-linear corrections calculated using the Zel'dovich approximation have the same self-similar behaviour.

We find that the scaling properties of the non-linear corrections to the two point correlation function are determined by the spatial behaviour of the linear pair velocity dispersion. For $\gamma < 2$ both the radial and tangential components of the pair velocity dispersion keep on increasing as $x^{2-\gamma}$ and it has a local dependence on the linear two point correlation function i.e. the linear pair velocity dispersion at some separation x depends only only the linear two point correlation at the same separation. As a consequence all the non-linear corrections to the two point correlation function also have a local dependence on the linear two point correlation function and the n th order non-linear correction to the correlation function $\xi^{(n)}(x,t)$ has the property $\xi^{(n)}(x,t) \sim [\xi^{(1)}(x,t)]^n$. We see that for these cases all the non-linear corrections have the same self-similar behaviour as linear two point correlation.

For $\gamma = 2$ the pair velocity dispersion increases logarithmically at large x and the n th order non-linear correction is of the form $\frac{[\ln(x)]^{n-1}}{x^{n\gamma}}$ i.e. $\xi^{(n)}(x,t) \sim [\ln\left(\frac{x}{x_1}\right)]^{n-1}[\xi^{(1)}(x,t)]^n$. Because of the extra $[\ln\left(\frac{x}{x_1}\right)]^{n-1}$ the non-linear corrections do not have the same self-similar behaviour as the linear two point correlation function.

For $\gamma > 2$ the pair velocity dispersion at large x reaches a constant value l^2 which is determined by the linear two point correlation at small scales. For this case the linear pair velocity dispersion has a non-local dependence on the linear two point correlation function. This also introduces a new length scale and the n th order correction is of the form $\frac{l^{2(n-1)}}{x^{\gamma+2(n-1)}}$. This has completely different scaling properties and the non-linear corrections do not have the same self-similar behaviour as the linear two pint correlation.

In an earlier paper (Bharadwaj 1996) we have studied the non-linear correction to the two point correlation function using perturbative gravitational dynamics. We only considered cases with $\gamma > 2$ and we found that the non-linear corrections do not have the same self-similar behaviour as the linear two point correlation function. We also found that this was due to the emergence of a new length scale l which appears here also and we interpreted this in terms of a simple diffusion process. Scoccimarro & Frieman (1996) have studied the lowest order non-linear correction to the power spectrum and the corresponding correlation functions include those with $\gamma < 2$. They find that for power spectra where the index n is less than -1 (or $\gamma < 2$), the lowest order non-linear correction has the same self-similar behaviour as the linear power spectrum and they also find that the self-similarity is broken for $n \geq -1$ (i.e.$\gamma \geq 2$). We see that ZA makes the same predictions as GD regarding the scaling properties of the non-linear corrections to the two point correlation function and we expect that any conclusions that can be drawn on the

basis of our investigations using ZA should also hold for GD. We find that in ZA the scaling properties of the non-linear corrections are decided by the spatial behaviour of the linear pair velocity dispersion. We propose that this is also true for the non-linear corrections calculated using perturbative gravitational dynamics. Although the relation between the non-linear corrections and the linear pair velocity dispersion is quite explicit in the Zel'dovich approximation (equation 8), it is not clear how this comes about in perturbative gravitational dynamics.

The pair velocity dispersion is the lowest velocity moment of the two point distribution function that has information about the velocity components tangential to the separation $\vec{x}$. It is very interesting that it is this quantity, and not some lower velocity moment, that turns out to play an important role in deciding the scaling properties of the non-linear corrections.

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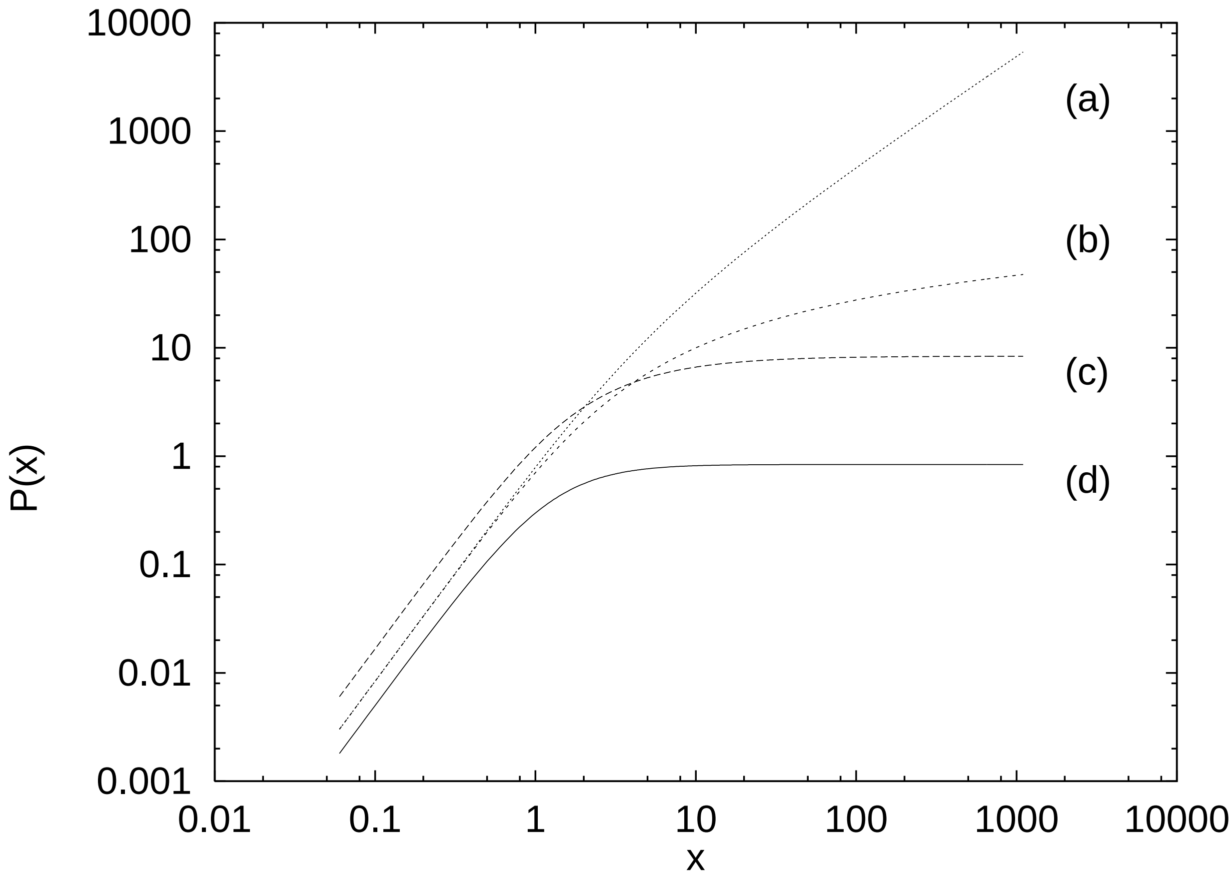

Fig. 1.— The tangential component of the pair velocity dispersion P(x) shown as a function of the separation x for the four cases discussed in section 3.4.